### Simple Harmonic Motion

#### Periodic Motion

A motion that repeats itself over and over is referred to as **periodic motion**. The period, $T$, is the time required for one cycle of periodic motion. The frequency, $f$, is the number of oscillations per unit time.

\[
 f = \frac{1}{T}
\]

#### Simple Harmonic Motion

The object has one position at which the net force on it is zero. At that position, the object is in equilibrium. Whenever the object is pulled away from its equilibrium position, the net force on the system becomes nonzero and pulls the object back toward equilibrium.
13.2 Simple Harmonic Motion

**Position Versus Time in Simple Harmonic Motion**

$x = A \cos(2\pi ft)$

- $x =$ position    
- $f =$ frequency    
- $A =$ amplitude    
- $t =$ time

2 Types of Harmonic Oscillators

**Spring Oscillator**

From the interactive, you may have noticed that a larger mass $m$ results in a larger period. On the other hand, a larger spring constant $k$ results in a smaller period.

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**The Period of a Mass on a Spring**

$$T = 2\pi \sqrt{\frac{m}{k}}$$

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**Hooke's Law**

$$F = -kx$$

The force exerted by a spring is equal to the spring constant times the distance the spring is compressed or stretched from its equilibrium position.

**Potential Energy in a Spring**

$$U = \frac{1}{2}kx^2$$

The potential energy in a spring is equal to one-half times the product of the spring constant and the square of the displacement.
The Pendulum

A simple pendulum consists of a mass \( m \) hanging from a string or rod of length \( L \) and fixed at a pivot point. When displaced to an initial angle and released, the pendulum will swing back and forth with simple harmonic motion.

From the interactive, you may have noticed that a larger length \( L \) results in a larger period. On the other hand, a larger acceleration due to gravity \( g \) results in a smaller period.

The Pendulum Wave

Unit 13 In-Class Problems

3. An air-track cart attached to a spring completes one oscillation every 2.4 s. At \( t = 0 \) the cart is released from rest at a distance of 0.10 m from its equilibrium position. What is the position of the cart at 2.7 s?

\[ T = 2.4 \text{ s} \]
\[ A = 0.10 \text{ m} \]
\[ t = 2.7 \text{ s} \]

\[ x = (0.10 \text{ m}) \cos (2\pi f t) \]

\[ x = (0.10 \text{ m}) \cos [2\pi (0.417 \text{ Hz})(2.7 \text{ s})] = 7.0 \text{ cm} \]

4. A 0.120 kg mass attached to a spring oscillates with an amplitude of 0.0750 m and a maximum speed of 0.524 m/s. Find (a) the force constant and (b) the period of motion.

5. The pendulum in a grandfather clock is designed to take one second to swing in each direction; 2.00 seconds for a complete period. Find the length of the pendulum that is required to keep the correct time.
4. A 0.120 kg mass attached to a spring oscillates with an amplitude of 0.0750 m and a maximum speed of 0.524 m/s. Find (a) the force constant and (b) the period of motion.

\[ f = kx \quad U_s = K_{max} \]

\[ K_{max} = U_{max} \]
\[ \frac{1}{2}mv^2 = \frac{1}{2}kx^2 \]
\[ k = 5.86 \text{ N/m} \]

\[ T = 2\pi \sqrt{\frac{m}{k}} \]
\[ T = 2\pi \sqrt{\frac{0.120 \text{ kg}}{5.86 \text{ N/m}}} \]
\[ T = 0.899 \text{ s} \]

5. The pendulum in a grandfather clock is designed to take one second to swing in each direction; 2.00 seconds for a complete period. Find the length of the pendulum that is required to keep the correct time.

\[ T = 2\pi \sqrt{\frac{L}{g}} \]
\[ L = \frac{gT^2}{4\pi^2} \]
\[ T^2 = 4\pi^2 \frac{L}{g} \]
\[ L = 0.994 \text{ m} \]
Unit 13 Problems
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